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Robust Adaptive Control of a Weakly Minimum Phase General Aviation Aircraft

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Abstract

Loss of control is the leading cause of accidents in General Aviation aircraft. Installation of autopilots on every airplane would significantly reduce this type of accident. Adaptive controls could offer an affordable autopilot for a wide variety of aircraft, by allowing the gains to automatically adapt. This investigation presents the design and test of an adaptive autopilot. The autopilot was designed to control the altitude and bank angle of a small airplane at a specific trim condition. The airplane was then moved to extreme points in its flight envelope, and the autopilot successfully adapted to maintain controller performance, except at airspeeds near stall. Since the aircraft is weakly minimum phase, a zero filter was used in the adaptive autopilot, which required a nominal model of the linearized plant at the design condition. Turbulence was added to test robustness, and the autopilot was able to maintain its performance.

Keywords: adaptive control; general aviation; autopilot design; loss of control

1. Introduction

In the period between 1998 and 2004, General Aviation (GA) accounted for more than half the total number of hours flown by civilian aircraft, with an accident rate of more than double that of commercial flights[1]. The Federal Aviation Administration (FAA) has identified the leading cause of these accidents to be Loss of Control (LOC). LOC is "a significant, unintended departure of an aircraft from controlled flight, the operational flight envelope, or usual

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flight attitudes, including ground events[1].” One of the most effective resources to prevent LOC accidents is the use of autopilots. The Aircraft Owners and Pilots Association estimates that night, instrument meteorological conditions, LOC accidents could be reduced by 50% if autopilots were installed in all IFR capable GA aircraft[2]. However, strict FAA regulations for the certification of autopilots make this solution unviable. Autopilot manufacturers must demonstrate, to a high degree of certainty, that the autopilot software will not jeopardize the safety of the aircraft for each make and model of aircraft they intend to install their product on. The high certification costs is ultimately transferred to the consumer, who ends up paying significantly more money for the certification process than for the actual product itself. Evidence of this fact can be found in autopilots for airplanes certified in the experimental category. These autopilots do not need to be certified and, therefore, cost a fraction of the price of their certified counterparts, while they are comprised of essentially the same hardware and software and provide nearly the same functionality.

Adaptive control theory offers a viable solution for this problem. By designing an adaptive autopilot, the need to make and model specific certification would disappear. The autopilot gains would adapt to different airframes while maintaining the same controller structure. If such generic autopilot could be demonstrated to be safe and reliable to a level of confidence acceptable by the FAA, the time and cost savings would be significant for manufacturers, operators, and regulating agencies alike. However, current regulations prevent the certification of nonlinear controllers. Technological advances in control theory have outpaced the certification process updates, leading to newer, safer technology being uncertifiable due to outdated regulations[3].

This investigation is aimed at demonstrating the viability of an adaptive controller as an autopilot for a GA aircraft. For this purpose, a direct adaptive controller was designed and tested in simulation at a given flight condition. The aircraft was then moved to extreme points in its flight envelope to demonstrate that a single set of adaptive gains can compensate for system uncertainty. Turbulence was added to test the robustness of the autopilot. The aircraft model used, and simulation environment are described in Section II. Section III provides a background on direct adaptive control theory, and Section IV presents the controller design. Section V contains the results, while Section VI contains the conclusions and recommendations for future work.

2. Simulation Environment

The aircraft model used for this investigation is a high fidelity, non-linear model of a Cirrus SR22. The SR22 is a single-engine general aviation aircraft with a gross take-off weight of 3,600 lb and a cruise speed of 183 kts. The aerodynamic database was obtained through a flight test program and model identification performed by the Eagle Flight Research Center at Embry-Riddle Aeronautical University. The model consists of the entire flight envelope of the aircraft, including non-linear regions, such as stall, and has been validated against FAA Part 60 tests. Fig. 1 shows a plot of the principal stability derivatives of the model.

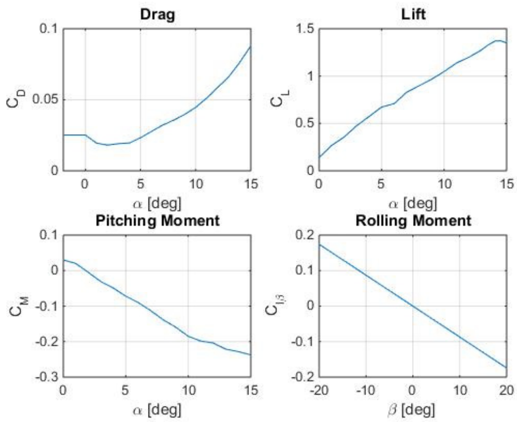

Fig. 1. Stability derivatives of the SR22 model used.

To simulate bounded disturbances, a turbulence model was used. The model used was the Dryden Wind Turbulence Model. In it, turbulence is created by passing band-limited white noise through forming filters. The model is based in the mathematical representation in the Military Specification MIL-F-8785C and it produces linear wind turbulence as well as rotational components.

3. Adaptive Controls Background

Consider the linear plant given by:

$$\begin{aligned} \dot{x} &= Ax + Bu + \Gamma u_D; x(0) = x_0 \in \Re^N \\ y &= Cx \end{aligned} \tag{1}$$

where $x(t) \in \Re^N$ is the plant state, $u(t)$, $y(t) \in \Re^M$ are the control input and plant output *m*-dimensional vectors respectively, and $u_D(t)$ is a disturbance with known basis functions $\phi_D(t)$.

The control objective is for the plant output, $y(t)$, to robustly asymptotically track the output $y_m(t)$ of a linear finite-dimensional Reference Model given by:

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m; x_m(0) = x_0^m \\ y_m &= C_m x_m \end{aligned} \tag{2}$$

where the reference model state $x_m(t)$ is an N_m-dimensional vector with reference model output $y_m(t)$ of the same dimension as the plant output $y(t)$. While the plant and reference model outputs must be of the same dimension, the reference model does not need to be of the same dimension as the plant. The reference model parameters will be completely known. "Robust asymptotic tracking" means that the output error vector, e_y, tends to a predetermined neighborhood of the vector zero, $N(0)$, as time tends to infinity. That is:

$$e_y \equiv y - y_m \xrightarrow[t \to \infty]{} N(0) \tag{3}$$

It has been demonstrated[4,5,6] that the control objective will be accomplished by a direct adaptive control law of the form:

$$u = G_e e_y + G_D \phi_D \tag{4}$$

with the adaptive gains, G_e and G_D, given by:

$$\begin{aligned} \dot{G}_e &= -aG_e - e_y e_y^* \gamma_e; \gamma_e > 0 \\ \dot{G}_D &= -aG_D - e_y \phi_D^* \gamma_D; \gamma_D > 0 \end{aligned} \tag{5}$$

if the Kalman-Yakubovich (K-Y) conditions are met. That is:

$$\exists P,Q>0 \text{ so that}$$
$$A^T P+PA=-Q \tag{6}$$
$$PB=C^T$$

It has been shown[7] that for a Single-Input-Single-Output (SISO) system, the K-Y conditions are equivalent to the system being minimum phase (no unstable zeros) and having a non-zero high frequency gain. This presents a problem in aircraft applications, since conventional configuration aircraft, where the horizontal tail is located aft of the center of gravity, are non-minimum phase in altitude. In addition, the angular rates contain a zero on the imaginary axis (marginally stable). This prevents the direct adaptive control law in (4) to be applied directly. However, below is a method for dealing with systems of this type[8].

If CB is nonsingular, then there exists an invertible, bounded linear operator W such that:

$$\begin{aligned}\bar{B} &\equiv WB=\begin{bmatrix} CB \\ 0 \end{bmatrix}\\ \bar{C} &\equiv CW^{-1}=\begin{bmatrix} I_m & 0 \end{bmatrix}\\ \bar{A} &\equiv WAW^{-1}\end{aligned} \tag{7}$$

This coordinate transformation can be used to put (1) into normal form:

$$\begin{aligned}\dot{y} &= \bar{A}_{11}y+\bar{A}_{12}z_2+CBu\\ \dot{z} &= \bar{A}_{21}y+\bar{A}_{22}z_2\end{aligned} \tag{8}$$

Where the subsystem: $(\bar{A}_{22},\bar{A}_{12},\bar{A}_{21})$ is called the *zero dynamics* of (1)[8]. The system in is *minimum phase* when $\bar{A}_{22}$ is stable. The system is *weakly minimum phase* when $\bar{A}_{22}$ can be rewritten, via a coordinate transformation, as $\begin{bmatrix} \bar{A}^u_{22} & 0 \\ 0 & \bar{A}^s_{22} \end{bmatrix}$ where $\bar{A}^s_{22}$ is stable and $\bar{A}^u_{22}$ has spectrum: $\sigma(\bar{A}^u_{22})=\{\lambda_1,\ldots,\lambda_l\}$ isolated unrepeated eigenvalues with $\mathrm{Re}(\lambda_k)=0$. For weakly non-minimum phase systems, the control law in (4) can be modified to include a zero filter to compensate for the unstable zero dynamics:

$$u=G_e e_y+G_D\phi_D+\hat{v}_u \tag{9}$$

with the following zero filter:

$$\begin{aligned}\hat{v}_u &= -(CB)^{-1}\bar{A}^u_{12}\hat{z}_u\\ \dot{\hat{z}} &= \bar{A}^u_{21}e_y+\bar{A}^u_{22}\hat{z}_u\end{aligned} \tag{10}$$

and adaptive gains defined as in (5). Then the adaptive gains will be bounded and the output tracking error, e_y, converges exponentially with rate e^{-at} to the ball of radius[8]:

$$R_* \equiv \frac{\left(1+\sqrt{p_{max}}\right)}{a\sqrt{p_{min}}} M_v \tag{11}$$

where $0 < a \leq \frac{q_{min}}{p_{max}}$, q_{min} is the smallest eigenvalue of the Q matrix defined in (6), p_{min} and p_{max} are the smallest and largest eigenvalues of the P matrix defined in (6), and M_v is the upper bound for the unknown disturbances, in this case the turbulence. These numerical values are usually estimated from a low-order approximate model of the aircraft.

4. Controller Design

The autopilot was designed by separating the longitudinal and lateral controllers. The longitudinal controller provides altitude tracking while the lateral controller provides bank angle tracking. A separate navigational controller provides the required bank and altitude commands for three dimensional navigation. The target performance was the performance of conventional autopilots, which is a rate of climb of 800 fpm and a rate of descent of 500 fpm for the longitudinal controller and a rate of turn of 80% standard rate turn for the lateral controller. The autopilot is required to perform its control duties even in the presence of turbulence, so robustness considerations were taken into consideration in the design of the control laws. The nonlinear model was linearized at a trim condition with an airspeed of 155 kts and an altitude of 3000 ft. The following subsections describe each of the separate controllers individually.

4.1. Longitudinal Controller

The primary objective of the longitudinal autopilot is to control the aircraft's altitude. In order to do this, three conventional controllers and an adaptive controller were connected in series. In this architecture, altitude is tracked

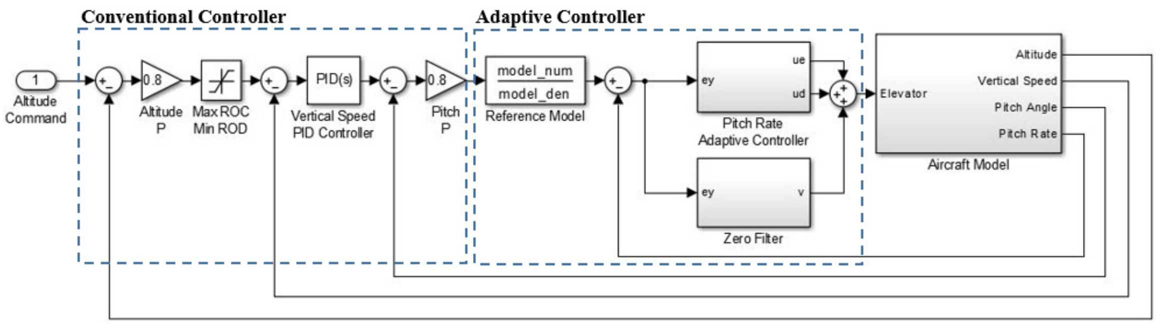

Fig. 2. Longitudinal controller architecture.

through vertical speed by the use of a proportional controller. Vertical speed is tracked through pitch angle by means of a PID controller. The pitch angle is tracked using pitch rate through the use of a proportional controller. Lastly, the desired pitch rate is tracked using a model reference direct adaptive controller with a zero filter. Disturbance rejection is performed by the adaptive controller on pitch rate. Fig. 2 shows the control architecture used.

4.1.1. Pitch rate controller

The linearized longitudinal model of the aircraft from elevator deflection to pitch rate is given below:

$$\begin{bmatrix} \dot{V}_{TAS} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -2.0492 & -0.0009 & 0 & 0.9664 \\ 40.5447 & -0.0221 & -32.1207 & 0 \\ 0 & 0 & 0 & 1 \\ -29.4071 & 0 & 0 & -4.8326 \end{bmatrix} \begin{bmatrix} V_{TAS} \\ \alpha \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0.0025 \\ 0 \\ 0 \\ 0.8355 \end{bmatrix} \delta_e$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{TAS} \\ \alpha \\ \theta \\ q \end{bmatrix} \tag{12}$$

This system contains a zero on the imaginary axis, so a zero filter is required for the use of direct adaptive control. The matrix W that transforms this system into normal form is given by:

$$W = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -0.002979 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

The coordinate transformation in (7) leads to the following zero dynamics matrices:

$$\begin{aligned} \bar{A}_{11} &= -4.9205 \\ \bar{A}_{12} &= \begin{bmatrix} -29.4071 & 0 & 0 \end{bmatrix} \\ \bar{A}_{21} &= \begin{bmatrix} 0.9750 \\ 0.1213 \\ 1 \end{bmatrix} \\ \bar{A}_{22} &= \begin{bmatrix} -1.9612 & -0.009 & 0 \\ 40.5447 & -0.0224 & -32.1207 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{14}$$

The zero filter is then assembled as in (10). The adaptive gains were chosen as:

$$\begin{aligned} \dot{G}_e &= -0.05G_e - e_y e_y^* \\ \dot{G}_D &= -0.05G_D - e_y \end{aligned} \tag{15}$$

Note that the disturbance basis function, ϕ_D, has been chosen as 1. This means that the disturbances are treated as a series of steps of unknown amplitudes. The reference model was chosen as a second order system with a natural frequency of 25 and a damping ratio of 1.

4.2. Lateral Controller

The primary objective of the lateral controller is to track a bank angle command. To achieve this, bank angle is tracked through roll rate using a PID controller. The desired roll rate is then tracked using a model reference direct adaptive controller with disturbance rejection. Fig. 3 shows the control architecture used.

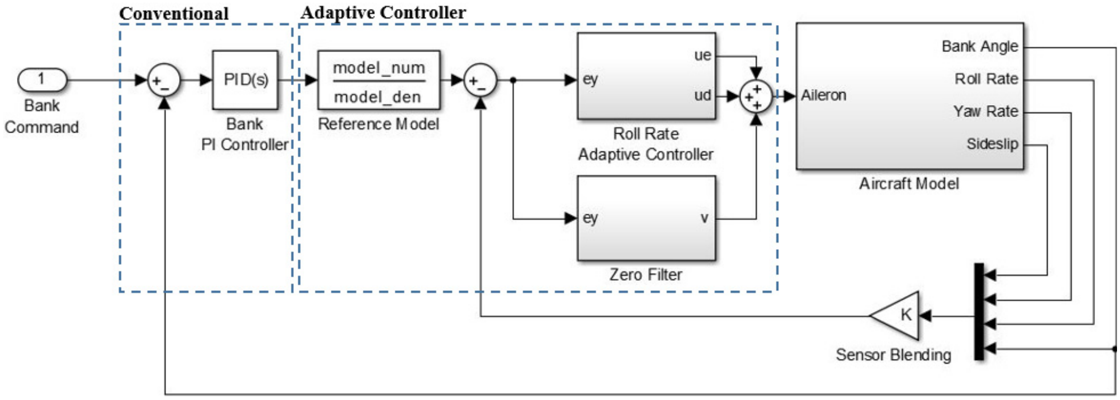

Fig. 3. Lateral controller architecture.

4.2.1. Roll rate controller

The linearized lateral-directional model of the aircraft from aileron deflection to roll rate is given below:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -0.2342 & 0.1221 & 0.0242 & -1.0196 \\ 0 & 0 & 1 & 0.0242 \\ 0 & 0 & -16.012 & 3.7012 \\ 6.6457 & 0 & 0.1134 & -0.4407 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.9415 \\ 0.0061 \end{bmatrix} \delta_a$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} \tag{16}$$

This system contains a zero that is slightly to the right of the imaginary axis, so sensor blending[9,10] was used to obtain the following blended output:

$$y = \begin{bmatrix} -0.0028 & 0.0034 & 1.0621 & -0.0004 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} \tag{17}$$

Note that this new blended output has a zero on the imaginary axis. The output is no longer strictly the roll rate, but the conventional outerloop controller compensates for the minimal discrepancy between the blended output and the actual roll rate.

The matrix W that transforms this system into normal form is given by:

$$W = \begin{bmatrix} -0.0028 & 0.0031 & 1.0621 & -0.004 \\ 1 & 0 & 0 & -0.0030 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -153.567 \end{bmatrix} \tag{18}$$

The coordinate transformation in (7) leads the following zero dynamics matrices:

$$\begin{aligned} \bar{A}_{11} &= -15.9854 \\ \bar{A}_{12} &= \begin{bmatrix} -36.5558 & 0.0493 & -0.0249 \end{bmatrix} \\ \bar{A}_{21} &= \begin{bmatrix} 0.0162 \\ 0.9471 \\ -31.0570 \end{bmatrix} \\ \bar{A}_{22} &= \begin{bmatrix} -0.2540 & 0.1221 & 0.006636 \\ 0.0026 & -0.0029 & -0.00016 \\ -1055.02 & 0.09639 & -0.4442 \end{bmatrix} \end{aligned} \tag{19}$$

The zero filter is then assembled as in (10). The adaptive gains were chosen as:

$$\begin{aligned} \dot{G}_e &= -0.05G_e - e_y e_y^* \\ \dot{G}_D &= -0.05G_D - e_y \end{aligned} \tag{20}$$

Note that, once again, the disturbance basis function, ϕ_D, has been chosen as 1. The reference model was chosen as a first order system with a time constant of 75.7. This is the same roll time constant as the original system.

5. Results

Although the controllers were designed for one flight condition, they were tested at 6 different flight conditions along the flight envelope. The airplane was trimmed for the particular flight conditions and then flown through a set of altitude and bank angle changes. The conditions tested are shown in Table 1.

Table 1. Trim conditions tested.

Trim Condition	Airspeed (*ft/s*)	Altitude (*ft*)
1	262	3000
2	140	500
3	160	8000
4	280	500
5	280	8000
6	200	3000

The altitude changes consisted of climbing 200 ft, then descending through 300 ft while performing two sets of bank angle doublets along the way. A proportional airspeed controller was used to prevent the airplane from stalling. The time histories for important parameters are shown in Fig 4. The altitudes and bank angle errors are shown instead of their actual values to compare all different trim conditions.

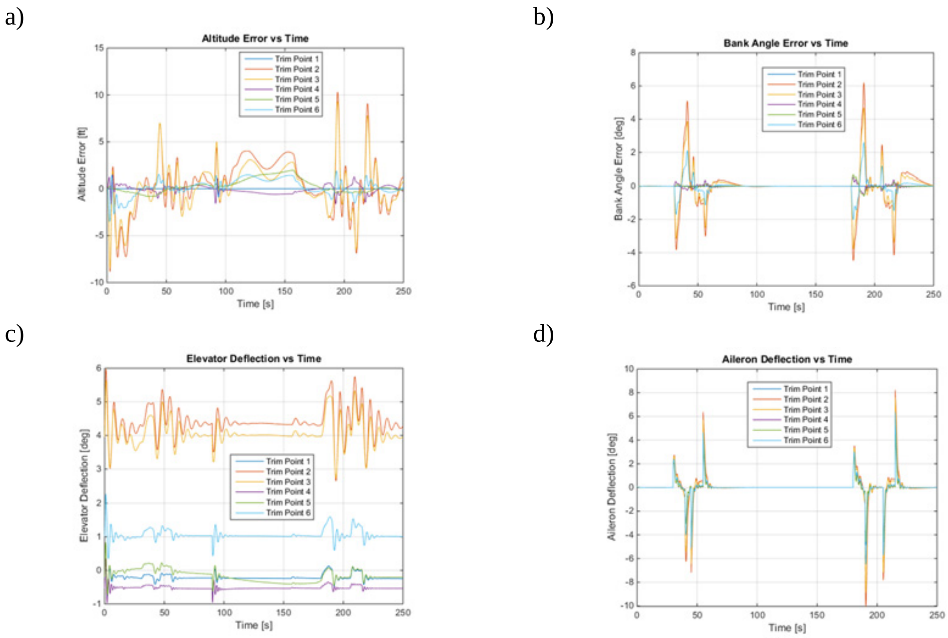

Fig. 4. a) Altitude error, b) bank angle error, c) elevator deflection, and d) aileron deflection time histories for different trim conditions

It can be seen in Fig. 4 that for trim conditions 1, 4, 5, 6, the altitude autopilot performance is nearly identical, while the performance of trim conditions 2 and 3 has been deteriorated while trying to maintain altitude. It is worth noting that trim conditions 2 and 3 are near the stall speed of the airplane (122 ft/s). It can also be seen that the bank angle tracking has remained nearly identical for all trim conditions tested. For both control surfaces, the deflection is reasonable and within the capabilities of the aircraft. The controls have not saturated and their rates are reasonable. However, the maximum deflection does vary depending on the trim condition (the lower the airspeed, the larger the maximum deflection). This reflects the loss of control effectiveness at lower airspeeds.

The next step was to include bounded disturbances. For this, the turbulence model was used. The results are shown in Fig. 5. It can be seen that the controller has been able to supress the persistent disturbances and completed its mission. The performance of the controller is equivalent to its performance in the absence of disturbances. The control deflections were still reasonable even in the presence of noise. However, the elevator deflection appears to be spending most of its effort in fighting the noise rather than controlling the aircraft. The aileron deflection is similar to the noise free case.

6. Conclusions

In conclusion, LOC is a serious problem for GA aircraft. Several mitigation strategies are currently in place. The installation of autopilots on every GA aircraft could further reduce loss of control accidents significantly. However, the certification costs of current autopilots render this option unviable. Adaptive control theory could provide an answer to this problem, but regulations prevent the use of nonlinear controllers due to their undeterministic nature.

This investigation was an initial effort toward a generic, adaptive autopilot.

a) b) c) d)

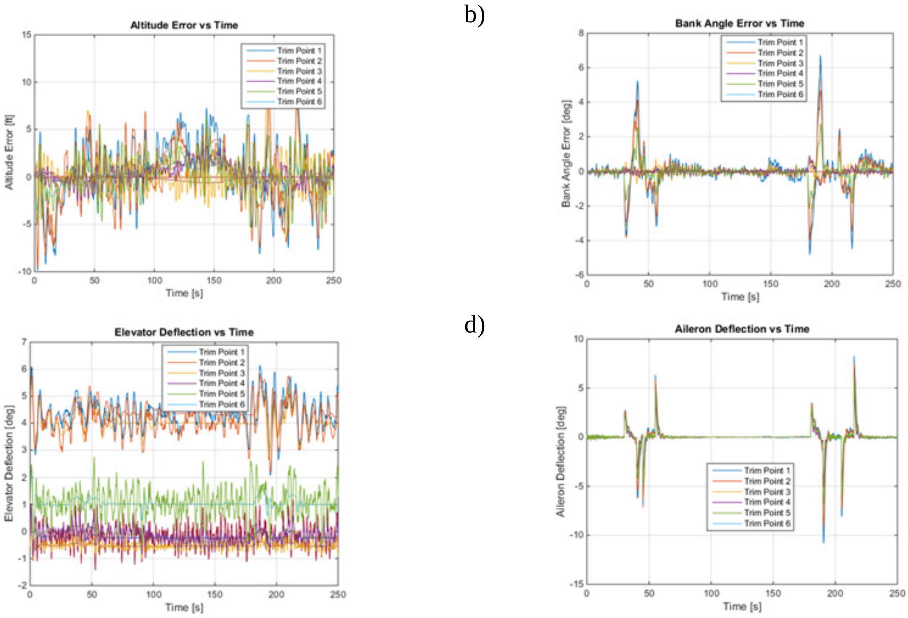

Fig. 5. a) Altitude error, b) bank angle error, c) elevator deflection, and d) aileron deflection time histories for different trim conditions with added noise.

An autopilot was designed for a GA airplane at a specified trim condition, and allowed to adapt as the aircraft flew at different points of its flight envelope. The autopilot was divided into longitudinal and lateral-directional modes and consisted of an adaptive inner loop and a conventional outer loop for each of the modes. Since the aircraft is weakly minimum phase, a zero filter was added in the adaptive portion of the controller. This zero filter required approximate knowledge of the linearized plant at the design condition. Furthermore, to test the robustness of the autopilot, atmospheric disturbances were added. It was concluded that the autopilot maintained its performance throughout the flight envelope even when operating in turbulence, except at airspeeds close to stall, where small oscillations started to occur.

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